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October 26, 2026
Due: November 2, 2026 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2026/27

Exercise Set 3

If you would like to submit your solutions for this problem set, please send them via email to jluz@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:
https://terminplaner.dfn.de/public/bookings/fbde794b-7dd7-4da6-a50d-328892905720
A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:
https://terminplaner.dfn.de/public/bookings/5fd62088-4476-43d6-ac4f-08905d9a9ec7

Exercise 1: (5 Points)
Consider the following randomized rounding for fractional Ski Rental. In step t, given that you have only rented skis before, buy the skis with probability $\frac{x_{\text{buy}}^{(t)} - x_{\text{buy}}^{(t-1)}}{1 - x_{\text{buy}}^{(t-1)}}$, otherwise rent them again. Show that if the underlying fractional algorithm is α-competitive so is the randomized integral algorithm.

Exercise 2: (4 Points)
We would like to show that optimal online algorithms that know the input distribution in advance are always deterministic.
To this end, we use the notation of Lecture 6. We have a finite class of deterministic algorithms $\mathcal{A}$ and a finite class of instances $\mathcal{X}$. Let X be a random variable with values in $\mathcal{X}$ and A be an *independent* random variable with values in $\mathcal{A}$. Show that there is an $a \in \mathcal{A}$ such that $\mathbf{E}[c(a, X)] \leq \mathbf{E}[c(A, X)]$.

Hint: Use the same techniques as in the proof of Yao's principle.

Exercise 3: (1+5 Points)
Consider the following online problem: We would like to buy a house. Each time we view a house, we get to know how good it is. Immediately after, we have to decide whether to buy the current house or not. If we do not buy it now, there will be no other opportunity later. Formally, this means that a sequence of values is revealed one after the other. Note that there are no assumptions concerning the order of the sequence. In particular, the order could be chosen by an adversary. The algorithm can always decide if it stops the sequence at its current value or if it waits. We say that an algorithm wins if it stops the sequence at its highest value.

(a) Assume that the length of the sequence n is known. Give a randomized online algorithm that wins with probability $\frac{1}{n}$.

(b) Use Yao's principle to show that no algorithm wins with higher probability.

Hint: Use the profit-maximization variant of Yao's principle. The profit is 1 if the algorithm wins, 0 otherwise. The offline optimum is clearly always 1.