

Algorithms and Uncertainty

Winter Term 2026/27

Exercise Set 2

If you would like to submit your solutions for this problem set, please send them via email to jluz@uni-bonn.de by Monday evening. Submitting solutions in groups is also possible.

If you would like to present one of your solutions in class, please use the following link to book a presentation slot by Monday evening:

<https://terminplaner.dfn.de/public/bookings/ac2f4bb2-64b1-416e-963c-1167aeae26c3>

A short meeting to discuss your solution is mandatory before presenting it in class. To book a time slot for this meeting, please use the following link by Monday evening as well:

<https://terminplaner.dfn.de/public/bookings/0a762dff-0007-429d-b2b6-95fa901cd7c3>

Exercise 1:

(3+4 Points)

Given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system.

- (a) Consider the unweighted version of Online Set Cover, i.e., $c_S = 1$ for all $S \in \mathcal{S}$, and the following algorithm: Upon arrival of element e , if $\sum_{S: e \in S} x_S = 0$, set $x_S = 1$ for all S with $e \in S$ and $y_e = 1$. Otherwise set $y_e = 0$. Show that this algorithm is f -competitive by using Lemma 3.7.
- (b) Now, we generalize the algorithm from (a) to the weighted version. Let $g_e = \max\{0, 1 - \sum_{S: e \in S} x_S\}$ and let S_e be the cheapest set covering e . Upon arrival of element e , for each S that covers e , increase x_S by $\frac{c_{S_e}}{c_S} g_e$ and set $y_e = c_{S_e} g_e$. Show that this algorithm is f -competitive by using Lemma 3.7.

Exercise 2:

(5 Points)

Again, given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system.

Use our results from the fourth lecture to devise an online algorithm that is $O(\log f)$ -competitive for fractional set cover and prove this. You may assume that f is known beforehand.

Hint: One bound in the analysis from the lecture can be improved for $f < m$. Use it to adapt the algorithm.