Thomas Kesselheim
Rebecca Lehming

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Algorithms and Uncertainty

Winter Term 2024/25

Tutorial Session - Week 14

Exercise 1:
Consider a zero-sum game between Anton and Betty in which the following matrix shows Bettys winnings:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} -4 & 2 & 5 \\ 2 & -4 & -3 \\ 3 & -6 & -2 \\ -3 & 8 & 6 \end{pmatrix} \end{array}$$

(a) Explain why Anton will never choose A_4. Use this fact to reduce the game to one involving a 3×3 matrix.

(b) Use similar thinking to reduce the game to one involving a 2×2 matrix.

(c) Solve the 2×2 matrix game found in (b). Then use your answer to reveal the optimal strategies for Anton and Betty in the original 4×3 game.

Exercise 2:
Under the assumption that there exists only a finite number of instances and a finite number of deterministic algorithms, show that there exists a randomized algorithm and a probability distribution over instances such that Yao's Prinicple applies with equality.