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Algorithms and Uncertainty

Winter Term 2024/25

Tutorial Session - Week 12

Exercise 1:
For the normed vector space $(\mathbb{R}^d, \|.\|)$, the unit ball with respect to $\|.\|$ is defined as the set $\{x \in \mathbb{R}^d : \|x\| \leq 1\}$.

(a) Show that the unit ball with respect to the 1-norm is convex.

(b) Show that the unit ball with respect to the 2-norm is convex.

(c) Show that the unit ball with respect to the ∞-norm is convex.

(d) Does the same result hold for an arbitrary p-norm with $p > 1$? What about $p < 1$?