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Algorithms and Uncertainty

Winter Term 2024/25

Tutorial Session - Week 10

Exercise 1:
Recall the regret definition from the lecture: $\text{Regret}^{(T)} = L_{\text{Alg}}^{(T)} - \min_i \sum_{t=1}^{T} \ell_i^{(t)}$.

(a) Use Yao's principle to show that for every (randomized) algorithm there is a sequence $\ell^{(1)}, \dots, \ell^{(T)}$ such that $L_{\text{Alg}}^{(T)} \geq \left(1 - \frac{1}{n}\right) T$ but $\sum_{t=1}^{T} \min_i \ell_i^{(t)} = 0$.

(b) Discuss the importance of the order of sum and minimum in the regret definition using your results from (a).

Exercise 2:
Show that every no-regret algorithm in the experts setting has to be randomized. Consider the case $n = 2$ and for every deterministic algorithm construct a sequence such that $L_{\text{Alg}}^{(T)} = T$ and $\min_i L_i^{(T)} \leq \frac{T}{2}$.