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Algorithms and Uncertainty

Winter Term 2024/25

Tutorial Session - Week 8

Exercise 1:
We want to consider the Stochastic Vertex Cover problem. The edge set $A \subseteq E$ is uncertain, but drawn from a known probability distribution. The probability that the edge set is $A \subseteq E$ is given by p_A. Our goal is to compute a Vertex Cover of minimum cost for the graph $G = (V, A)$. Before A is revealed, we have to pay c_v^{I} for v, afterwards $c_v^{\mathrm{II}} \geq c_v^{\mathrm{I}}$.

(a) Derive an LP such that every policy corresponds to a feasible solution. Consider variables x_v denoting if v is picked in the first stage and $y_{A,v}$ if the edge set is A and v is picked in the second stage.

(b) In order to compute a feasible policy, we use the following algorithm which uses an optimal LP solution (x^*, y^*) of the LP above:

1. In the first stage, pick all vertices for which $x_v^* \geq \frac{1}{4}$.
2. In the second stage, when knowing A, pick all vertices for which $y_{A,v}^* \geq \frac{1}{4}$.

Show that this algorithm always computes a feasible policy.

(c) Show that the expected cost of the computed policy is at most 4-times the expected cost of the optimal policy.