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January 20, 2025
Due: January 27, 2025 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2024/25

Exercise Set 14

If you want to hand in your solutions for this problem set, please send them via email to rlehming@uni-bonn.de *by Monday evening. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *stating **which task** you would like to present in **which of the tutorials**. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated on a first-come-first-serve basis, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (4 Points)

Let $G = (V, E)$ be a graph with edge capacities $(c_e)_{e \in E}$, a source $s \in V$ and a sink $t \in V$. Let $\mathcal{P}$ be the set of all s-t paths in G and $|\mathcal{P}| \leq m = |E|$. Show that, if $T \geq \frac{4}{\epsilon^2} |\mathcal{P}| \log m$, the algorithm from lecture 24 then guarantees $\sum_{P \in \mathcal{P}} x_P \geq (1 - \epsilon) F^*$ when using Multiplicative Weights as the experts algorithm with $\eta = \frac{\epsilon}{2}$.

Exercise 2: (0 Points)

We are in the Bandits with Knapsacks setting.

(a) Suppose we define the regret as

$$\text{Regret}^{(T)} := \max_{i=1,\dots,n} \sum_{t=1}^{T} r_i^{(t)} - \sum_{t=1}^{T} r_{I_t}^{(t)},$$

where I_t is the action we choose in step t. Show that the regret is unbounded.

(b) Suppose we now compare ourselves to the best action $i \in N$ in hindsight, where $N := \{i = 1, ..., n | \sum_{t=1}^{T} c_{i,j}^{(t)} \leq B \, \forall j\}$. What can we say about the regret in this case?