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October 14, 2024
Due: October 21, 2024 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2024/25

Exercise Set 2

If you want to hand in your solutions for this problem set, please send them via email to rlehming@uni-bonn.de *by Monday evening. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *stating* ***which task*** *you would like to present in* ***which of the tutorials****. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated on a first-come-first-serve basis, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (3+4 Points)

Given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the *frequency* of the set system.

(a) Consider the unweighted version of Online Set Cover, i.e., $c_S = 1$ for all $S \in \mathcal{S}$, and the following algorithm: Upon arrival of element e, if $\sum_{S:e \in S} x_S = 0$, set $x_S = 1$ for all S with $e \in S$ and $y_e = 1$. Otherwise set $y_e = 0$. Show that this algorithm is f-competitive by using Lemma 3.7.

(b) Now, we generalize the algorithm from (a) to the weighted version. Let $g_e = \max\{0, 1 - \sum_{S:e \in S} x_S\}$ and let S_e be the cheapest set covering e. For each S that covers e, increase x_S by $\frac{c_{S_e}}{c_S} g_e$ and set $y_e = c_{S_e} g_e$. Show that this algorithm is f-competitive by using Lemma 3.7.

Exercise 2: (5 Points)

Again, given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system.

Use our results from the fourth lecture to devise an online algorithm that is $O(\log f)$-competitive for fractional set cover and prove this. You may assume that f is known beforehand.

Hint: One bound in the analysis from the lecture can be improved for $f < n$. Use it to adapt the algorithm.