Thomas Kesselheim
Rebecca Lehming

October 7, 2024
Due: October 14, 2024 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2024/25

Exercise Set 1

If you want to hand in your solutions for this problem set, please send them via email to rlehming@uni-bonn.de *by Monday evening. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to rlehming@uni-bonn.de *stating* ***which task*** *you would like to present in* ***which of the tutorials****. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated on a first-come-first-serve basis, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (1+3 Points)

Consider the following algorithm for the ski rental problem: Buy a pair of skis on your first skiing day for a price of B.

(a) Give a sequence σ of skiing/non-skiing days such that $c(\text{ALG}(\sigma)) = c(\text{OPT}(\sigma))$.

(b) Is there an $\alpha > 0$ such that this algorithm is strictly α-competitive? If yes, give a proof, otherwise a counterexample.

Exercise 2: (3 Points)

In an undirected graph $G = (V, E)$ a Vertex Cover is defined as a set $C \subseteq V$, where $e \cap C \neq \emptyset$ for all $e \in E$.

The Online Bipartite Vertex Cover problem is defined as follows: We are given a bipartite graph with vertices $V = L \cup R$. The vertices in L are offline which means they are present initially. Vertices in R are online and revealed one at a time together with their incident edges. Each vertex $v \in V$ has a cost c_v. At any point in time we need to maintain a Vertex Cover C on the present graph. We may only add but not remove vertices from C. The goal is to minimize the total cost $\sum_{v \in C} c_v$.

Consider the ski rental problem in a simplified version, i.e. we assume that every day is a skiing day but we do not know the number of days in advance. Show in a constructive way that the ski rental problem is a special case of the Online Bipartite Vertex Cover problem.

Exercise 3: (4 Points)

We want to show that the assumptions (completeness and triangular inequality) in the lecture concerning the online Steiner tree problem are indeed without loss of generality. Therefore, consider an α-competitive online algorithm for the online Steiner tree problem on complete graphs satisfying the triangular inequality (a.k.a. metric Steiner tree problem). Show how to convert this algorithm into an α-competitive one for the general online Steiner tree problem.