

## Algorithms and Uncertainty

Winter Term 2023/24

Tutorial Session - Week 10

### Exercise 1:

Recall the regret definition from the lecture:  $\text{Regret}^{(T)} = L_{\text{Alg}}^{(T)} - \min_i \sum_{t=1}^T \ell_i^{(t)}$ .

We want to understand the order of minimum and sum in the second term. Therefore, work on the following tasks.

- (a) Use Yao's principle to show that for every (randomized) algorithm there is a sequence  $\ell^{(1)}, \dots, \ell^{(T)}$  such that  $L_{\text{Alg}}^{(T)} \geq (1 - \frac{1}{n}) T$  but  $\sum_{t=1}^T \min_i \ell_i^{(t)} = 0$ .
- (b) Discuss the importance of the order of sum and minimum in the regret definition using your results from (a).

### Exercise 2:

Show that every no-regret algorithm in the experts setting has to be randomized. Consider the case  $n = 2$  and for every deterministic algorithm construct a sequence such that  $L_{\text{Alg}}^{(T)} = T$  and  $\min_i L_i^{(T)} \leq \frac{T}{2}$ .