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January 15, 2024
Due: January 22, 2024 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2023/24

Exercise Set 13

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Monday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (3 Points)
Prove Observation 24.4: If R is σ-strongly convex and $f_1, f_2, \dots$ are convex then $R + \sum_t f_t$ is σ-strongly convex.

Exercise 2: (4 Points)
We consider Online Linear Regression as introduced in the lecture. Recall that

$$f_t(w_1, w_2) = \left(w_1 x^{(t)} + w_2 - y^{(t)}\right)^2 \quad .$$

Derive a regret bound for Follow-the-Regularized-Leader with Euclidean regularization under the assumption that $|x^{(t)}|, |y^{(t)}| \leq 1$ for all t and $S = \{\mathbf{w} \in \mathbb{R}^2 \mid \|\mathbf{w}\|_2 \leq r\}$.

Exercise 3: (4 Points)
Derive a regret bound for Follow-the-Regularized-Leader if the Lipschitz constant depends on the time step, that is,

$$f_t(\mathbf{u}) - f_t(\mathbf{v}) \leq L_t \|\mathbf{u} - \mathbf{v}\| \quad \text{for all } \mathbf{u}, \mathbf{v} \in S \ .$$