Thomas Kesselheim
Anna Heuser

December 18, 2023
Due: January 8, 2024 at 10:00 p.m.

Algorithms and Uncertainty

Winter Term 2023/24

Exercise Set 11

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Monday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Monday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Monday evening is highly recommended.*

Exercise 1: (5 Points)

State a no-regret algorithm for the case that $\ell_i^{(t)} \in [-\rho, \rho]$ for all i and t. Also give a bound for the regret. You should reuse algorithms and results from the lectures.

Exercise 2: (5 Points)

We consider a different form of feedback. After step t, the algorithm does not get to know $\ell_i^{(t)}$ for all i but a noisy version. More precisely, an adversary first fixes the sequence $\ell^{(1)}, \ldots, \ell^{(T)}$, where all costs are in $[0, 1]$. Afterwards, from this sequence $\bar{\ell}^{(1)}, \ldots, \bar{\ell}^{(T)}$ is computed, where $\bar{\ell}_i^{(t)} = \ell_i^{(t)} + \nu_i^{(t)}$ and $\nu_i^{(t)}$ is an independent random variable on $[-\epsilon, \epsilon]$ with $\mathbf{E}[\nu_i^{(t)}] = 0$.
State a no-regret algorithm and a bound for the regret. You can make use of the previous exercise and the ideas presented in lecture 20.

Exercise 3: (3 Points)

In the lecture, we used that $\mathbf{E}\left[\min_i \sum_{t=1}^T \ell_i^{(t)}\right] \leq \min_i \mathbf{E}\left[\sum_{t=1}^T \ell_i^{(t)}\right]$ or $\mathbf{E}\left[\max_i \sum_{t=1}^T r_i^{(t)}\right] \geq \max_i \mathbf{E}\left[\sum_{t=1}^T r_i^{(t)}\right]$ respectively. Give a proof of this inequality.