Thomas Kesselheim
Alexander Braun

January 27, 2022
Due date: February 2, 2022 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 14

Exercise 1: (3 Points)

The *participation criterion* requires that the addition of a voter who strictly prefers candidate A to B should not change the winner from candidate A to candidate B.

Prove that plurality voting satisfies the participation criterion.

Exercise 2: (4 Points)

Again, consider the *participation criterion* from the previous task which requires that the addition of a voter who strictly prefers candidate A to B should not change the winner from candidate A to candidate B.

Show that instant runoff voting does not satisfy the participation criterion.

Hint: You may do this in two steps: First, state a counterexample in which you add a couple of voters in order to show that the property is violated. Then, you can think of sequentially adding those voters to the original instance and detecting the point in which the winner changes from candidate A to B.

Exercise 3: (2+2 Points)

Consider a set of miners who have discovered large bars of gold. The value of the loot to the group is the number of bars that they can carry home. It takes two miners to carry one bar, and thus the value of the loot to any subset of k miners is $\lfloor k/2 \rfloor$. Prove the following statements:

(a) If k is even, then there is a vector ψ which is in the core.

(b) If k is odd, then the core can be empty.

Remark: It might be convenient to consider a payoff-maximization variant of the ideas introduced in Lecture 26 instead of working in the cost-sharing environment.