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Due date: January 26, 2022 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 13

Exercise 1: (3 Points)
Prove that the men-proposal algorithm is not DSIC for the right-hand side (the women). For this purpose, give an instance of the stable matching problem in which, by lying about her preferences during the execution of the men-proposing algorithm, a woman can end up with a man that she prefers over the man she would have ended up with had she told the truth.

Exercise 2: (4 Points)
Show that there are instances of the stable matching problem in which the Gale-Shapley Algorithm (men-proposing algorithm) runs for $\Omega(n^2)$ iterations before terminating (with a stable matching). For this purpose, state an instance of the problem depending on n with suitable chosen preference orders and lower bound the number of iterations of the algorithm.

Hint: Consider an instance with $|U| = |V| = n$. Try to enforce exactly one rejection per iteration.

Exercise 3: (3 Points)
Recall the setting for Cake Cutting from Lecture 24. Show that if valuations are identical, i.e. $v_i(.) = v_j(.)$ for all $i, j \in N$, then the notions of Proportionality, Envy-Freeness and Equitability coincide.

Exercise 4: (3 Points)
Consider the algorithm (which is also known as the *moving-knife algorithm*) given in Section 4 of Lecture 24 that determines a proportional allocation for any number of agents n.

Show that even in the case of three agents the allocation of the algorithm might not be envy-free.