Algorithmic Game Theory
Winter Term 2021/22
Exercise Set 12

Exercise 1: (3 Points)

Consider the problem of Pairwise Kidney Exchange by Matching from Lecture 22. We want to show that the given matching algorithm is not DSIC if the order in which the algorithm processes the agents depends on their reports. For this purpose, consider a modified algorithm that processes agents in ascending order of node degree (tie-breaking in favor of the agent with the smallest index) and verify that there exists an instance such that there exists an agent who can do better by misreporting.

Exercise 2: (5 Points)

Consider a set of n teams, each with 10 players, where each team owner has a ranking of all $10n$ players. Define a notion of *stable allocation* in this setting (as in Definitions 22.1 and 22.2) and show how to adapt the top trading cycle algorithm to find a stable allocation. We assume that players' preferences play no role.

Exercise 3: (4 Points)

We call an allocation π *weakly stable* if there exists no set of agents who can obtain better houses than they are assigned in π by reallocating among themselves the houses allocated to them in π. Show that weak stability follows from stability as defined in Section 1 of Lecture 22.

Note: The converse does not hold. For example, if there are two agents who both prefer the same house, the only stable allocation is to give that house to its owner, but the alternative is also weakly stable.