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December 23, 2021
Due date: January 12, 2022 at 10:00 a.m.

Algorithmic Game Theory

Winter Term 2021/22

Exercise Set 11

We wish you a restful festive period, a Merry Christmas and a healthy, happy New Year 2022.

Exercise 1: (2+2+2 Points)

Determine the virtual value function φ of the following probability distributions.

(a) Uniform distribution on the interval $[a, b]$.

(b) Exponential distribution with rate $\lambda > 0$ (defined on $[0, \infty)$).

(b) The distribution given by the cumulative distribution function $F(v) = 1 - \frac{1}{(v+1)^c}$ defined on the interval $[0, \infty)$, where $c > 0$ is considered to be an arbitrary constant.

Which of the stated distributions are regular?

Exercise 2: (1+3 Points)

Once again, consider a single-item auction with two bidders whose valuations are drawn independently from a uniform distribution over $[0, 1]$.

(a) Prove that the random variables $\varphi_i(v_i)$ are distributed according to a uniform distribution on $[-1, 1]$.

(b) Utilize subtask (a) and the results of the lecture in order to determine the expected revenue of a second-price auction with reserve price $p \in [0, 1]$.

Exercise 3: (2+2+2 Points)

We want to discuss non-truthful mechanisms. Therefore, consider a single-item first-price auction with n bidders whose values are drawn uniformly at random from $[0, 1]$.

(a) Show that each bidder reporting a $\frac{n-1}{n}$-fraction of their actual value is a Bayes-Nash equilibrium.

(b) Compute the expected revenue of the first-price auction at equilibrium.

(c) Compare the expected revenue of the first-price auction at equilibrium to the one of a second-price auction (without reserve price). What do you recognize?