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Algorithmic Game Theory

Winter Term 2021/22

Tutorial Session - Week 11

Exercise 1:
Consider the following single-item auctions with two bidders whose valuations are drawn independently from a uniform distribution on the interval $[0, 1]$. Do not make use of the results of Lecture 20 in order to solve subtasks (a) and (b):

(a) Show that the expected revenue of a second-price auction is $\frac{1}{3}$.

(b) Now, define a second-price auction with *reserve price* p. Let v_1 and v_2 be the valuations of the bidders. The allocation and payment rule will be determined according to the following cases:

1. $\min\{v_1, v_2\} \geq p$: Like in the second price auction.
2. $\max\{v_1, v_2\} < p$: Nobody gets the item and no payments.
3. $v_1 \geq p > v_2$: Bidder 1 gets the item and has to pay p.
4. $v_2 \geq p > v_1$: Analogous to 3.

Show that using a reserve price of $\frac{1}{2}$ the second-price auction generates an expected revenue of $\frac{5}{12}$.

Additional Task: Can you get the same results by the use of virtual values? Calculate the expected revenue of (a) and (b) using the results of Lecture 20.