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Algorithmic Game Theory

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Tutorial Session - Week 6

Exercise 1:
Consider the following single-item auction with $n \geq 2$ bidders. The bidders simultaneously submit their bids $b_i \geq 0$. However, the item will always be allocated to the bidder with index 1 and the mechanism will make him/her pay the bid of the bidder with index 2.

(a) Show that the described mechanism is truthful.

(b) We call a mechanism *individually rational* if for all bidders $i \in \mathcal{N}$ bidding truthfully against an arbitrary bid profile of the other players never leads to a negative utility: If $v_i(x) \geq 0$ for all allocations $x \in X$, then $u_i((v_i, b_{-i}), v_i) \geq 0$.

Show that the given mechanism is not individually rational.

Exercise 2:
Analogous to the auctions that we defined in the lecture, we will consider the following *Third-Price Auction*. Just like in the first- and second-price auctions, bidders simultaneously submit their bids $b_i \geq 0$ and the winner will be determined as the bidder with the highest bid. Finally, the mechanism will make him/her pay the third highest bid. Prove that the described mechanism is not truthful.