

Algorithmic Game Theory

Winter Term 2020/21

Tutorial Session - Week 10

Exercise 1:

Consider the following single-item auctions with two bidders whose valuations are drawn independently from a uniform distribution on the interval $[0, 1]$. Do not make use of the results of Lecture 19 in order to solve subtasks (a) and (b):

- (a) Show that the expected revenue of a second-price auction is $\frac{1}{3}$.
- (b) Now, define a second-price auction with *reserve price* p . Let v_1 and v_2 be the valuations of the bidders. The allocation and payment rule will be determined according to the following cases:
 1. $\min\{v_1, v_2\} \geq p$: Like in the second price auction.
 2. $\max\{v_1, v_2\} < p$: Nobody gets the item and no payments.
 3. $v_1 \geq p > v_2$: Bidder 1 gets the item and has to pay p .
 4. $v_2 \geq p > v_1$: Analogous to 3.

Show that using a reserve price of $\frac{1}{2}$ the second-price auction generates an expected revenue of $\frac{5}{12}$.

Additional Task: Can you get the same results by the use of virtual values? Calculate the expected revenue of (a) and (b) using the results of Lecture 19.