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Algorithmic Game Theory

Winter Term 2020/21

Exercise Set 11

Exercise 1: (3 Points)

Prove that the men-proposal algorithm is not DSIC for the right-hand side (the women). For this purpose, give an instance of the stable matching problem in which, by lying about her preferences during the execution of the men-proposing algorithm, a woman can end up with a man that she prefers over the man she would have ended up with had she told the truth.

Exercise 2: (2 Points)

Show that there is not necessarily a solution to the stable roommates problem: In this problem, there is a set of $2n$ people, each with a total preference order over all the remaining people. A matching of the people (each matched pair will become roommates) is stable if there is no pair of people that are not matched that prefer to be roommates with each other over their assigned roommate in the matching.

Exercise 3: (4 Points)

Show that there are instances of the stable matching problem in which the Gale-Shapley Algorithm (men-proposing algorithm) runs for $\Omega(n^2)$ iterations before terminating (with a stable matching). For this purpose, state an instance of the problem depending on n with suitable chosen preference orders and lower bound the number of iterations of the algorithm.

Hint: Consider an instance with $|U| = |V| = n$. Try to enforce exactly one rejection per iteration.