

## Algorithmic Game Theory

Winter Term 2020/21

Exercise Set 7

*We wish you a restful festive period, a Merry Christmas and a healthy, happy New Year 2021.*

**Exercise 1:** (3 Points)

Recall the auction of  $k$  identical items from the previous exercise sets: Each bidder can acquire at most one of the items. If bidder  $i$  gets one of the items, she has a value of  $v_i$ . Otherwise, that is, if she does not get an item, she has a value of 0.

Make use of the VCG-results from the lecture in order to design a truthful mechanism for this auction. For this purpose, explicitly state the function  $f$  and calculate the payment rule  $p$ .

**Exercise 2:** (4 Points)

As seen in the lecture, let  $f: V \rightarrow X$  be a function that maximizes declared welfare, i.e.,  $f(b) \in \arg \max_{x \in X} \sum_i b_i(x)$  for all  $b \in V$ . For each  $i$ , let  $h_i$  be an arbitrary function  $b_{-i} \mapsto h_i(b_{-i})$  which does not depend on  $b_i$ . We define a mechanism  $\mathcal{M} = (f, p)$  by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j \neq i} b_j(f(b)) .$$

Prove that  $\mathcal{M}$  is a truthful mechanism.

**Exercise 3:** (4 Points)

An *all-pay auction* is a single-item auction defined in almost the same manner as a first-price auction: Each bidder reports a bid  $b_i \geq 0$ . The bidder with the highest bid wins the item. However, every bidder must pay their own bid regardless of whether they win the item or not.

Be inspired by the steps of Section 2 in the notes of Lecture 14 to derive the symmetric Bayes-Nash equilibrium of an all-pay auction with  $n$  bidders and identical distributions.