Algorithmic Game Theory
Winter Term 2020/21
Exercise Set 6

Exercise 1:   (3 Points)

Recall the auction of k identical items (Homework Set 5, Exercise 2): each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0.

Make use of Myerson's Lemma in order to design a mechanism that is truthful. For this purpose, explicitly state the function f, verify that it is monotone, and calculate the payment rule p resulting from the integral formula.

Exercise 2:   (3+1 Points)

A billionaire considers selling tours to the moon. The cost of building a rocket is C. Let $N = \{1, \ldots, n\}$ be the set of people who initially have declared an interest in the trip. The billionaire wishes to design a mechanism that will recover his cost but does not have information about the private valuation the bidders have for joining the trip. Therefore, he runs the following auction given as pseudocode:

- All bidders $i \in N$ simultaneously submit their bids $b_i \geq 0$.

- $S \leftarrow N$

- While $S \neq \emptyset$ do

    - $S' \leftarrow \{i \in S \mid b_i \geq \frac{C}{|S|}\}$

    - If $S' = S$, then allocate a seat for each $i \in S$ and no seat for each $i \in N \setminus S$. All bidders $i \in S$ have to pay $\frac{C}{|S|}$. The rest of the bidders $i \in N \setminus S$ has to pay nothing. Return.

    - Otherwise, $S \leftarrow S'$

- Do not allocate any seat and charge no payments at all. Return.

(a) Show that the described mechanism is truthful.

(b) Show that if the bidders are truthful, the auction finds the largest set of bidders that can share the target cost C equally, if there is one.

Exercise 3: (5 Points)

Recall the *Greedy-by-Value* and *Greedy-by-Sqrt-Value-Density* algorithms for single-minded CAs of lecture 12. Let us analyse another greedy algorithm that looks as follows.

Greedy-by-Value-Density

- Re-order the bids such that $\frac{b_1^*}{|S_1^*|} \geq \frac{b_2^*}{|S_2^*|} \geq \cdots \geq \frac{b_n^*}{|S_n^*|}$.

- Initialize the set of winning bidders to $W = \emptyset$.

- For $i = 1$ to n do: If $S_i^* \cap \bigcup_{j \in W} S_j^* = \emptyset$, then $W = W \cup \{i\}$.

Let $d = \max_{i \in \mathcal{N}} |S_i^*|$. Show that the given algorithm yields a d-approximation.