MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20
Assignment 13

Deadline: **28** January before noon (To be discussed: **28/29**. January 2020)

1 Shatter function lemma

Show that the shatter function lemma is tight. That is, for all δ and n construct a range space of VC dimension δ on n points with $\Phi_\delta(n)$ sets.

2 The ε-net theorem

a) In the proof of the ε-net theorem, show that $Pr[E_1] \leq \Phi_\delta(2s) \cdot 2^{-\frac{s}{2r}}$.

b) Show that for a sufficiently large constant c, a random sample of size $s = c \cdot \delta \cdot r \log \left(\frac{r}{\phi} \right)$ is a $\frac{1}{r}$-net with probability $1 - \phi$.

3 Range spaces defined by graphs

Let $G = (V, E)$ be an undirected graph.

a) Let $(V, \mathcal{R})$ be the range space where $S \in \mathcal{R}$ is the set of vertices on a shortest path in G. Show that the VC dimension is 2 if shortest paths are unique.

b) Let $(V, \mathcal{N})$ be the range space where $\mathcal{N} = \{N_G(v) : v \in V(G)\}$ is the system of vertex neighborhoods (where $N_G(v) = \{u \in V(G) : \{u, v\} \in E(G)\}$). Show that the VC dimension is bounded if G is planar.

4 Dual range spaces

Analyze the VC dimension of the dual range spaces of

a) halfspaces in $\mathbb{R}^d$,

b) disks in $\mathbb{R}^2$.