MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20

Assignment 9

Deadline: **10** December before noon (To be discussed: **10/11**. December 2019)

1 Arrangements of Hyperplanes

a) Count the number of faces of dimensions 1 and 2 for a simple arrangement of n planes in $\mathbb{R}^3$.

b) Express the number of k-faces in a simple arrangement of n hyperplanes in $\mathbb{R}^d$.

2 Vertices of level at most k

a) Consider n lines in the plane in general position (their arrangement is simple). Call a vertex v of their arrangement an *extreme* if one of its defining lines has a positive slope and the other one has a negative slope. Prove that there are at most $O(k^2)$ extremes of level at most k. Imitate the proof of Clarkson's theorem on levels.

b) Show that the bound in a) cannot be improved in general.

3 Intersections of circles of level at most k

Let $K_1, \ldots, K_n$ be circular disks in the plane. Show that the number of intersections of their boundary circles that are contained in at most k disks is bounded by $O(nk)$. Assume general position if convenient.

4 Exponential function

Verify that $1 - x \geq \mathrm{e}^{-2x}$, for any $x \in \left[0, \frac{1}{2}\right]$.