MA-INF 1203 Discrete and Computational Geometry

Wintersemester 2019/20

Assignment 8

Deadline: **3** December before noon (To be discussed: **3/4**. December 2019)

1 Crossing numbers

a) Show that for any n and m, $5n < m < \binom{n}{2}$, there exist graphs with n vertices, m edges, and crossing number $O(m^3/n^2)$.

b) Prove that in a drawing of G with the smallest possible number of crossings, no two arcs intersect more than once (including intersections at their endpoints).

2 Incidences

By extending the example which shows $I(n, n) = \Omega(n^{4/3})$, prove that for all m, n with $n^2 \geq m$ and $m^2 \geq n$, we have $I(m, n) = \Omega(n^{2/3}m^{2/3})$.

3 Cuttings

a) Show that if we don't assume general position, then for any $n, r \in \mathbb{N}$, with $r \leq n$, there is a set of n lines in the plane which admits an $\frac{1}{r}$-cutting with $O(r)$ (generalized) triangles.

b) Consider an arrangement $\mathcal{A}$ of n lines in the plane, in general position. Calculate the total number of (generalized) triangles arising by partitioning each cell of $\mathcal{A}$ into (generalized) triangles by adding suitable diagonals.