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Advanced Algorithms
WS 2019/20
Homework 3

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Exercise 1:

Let $G = (V, E)$ be a directed graph. What could be the change of the number of strongly connected components of G if we delete (insert) an edge?

Exercise 2:

Let $G = (V, E)$ be a directed graph. Let $G_{red} = (V_{red}, E_{red})$ be the reduced graph of G defined as in the lecture. Develop a linear time algorithm for the computation of the reduced graph for a given directed graph G. Take care that E_{red} does not contain multiple edges. Prove the correctness of the algorithm.

Exercise 3:

The *transitive closure* of a directed graph $G = (V, E)$ is a directed graph $H = (V, E')$ such that for $v, w \in V$, the edge $(v, w) \in E'$ iff there is a path from v to w in G. Develop an algorithm for the computation of the transitive closure of a given directed graph. What is the time used by your algorithm?

Exercise 4:

A directed graph $G = (V, E)$ is called *half connected* if for every $u, v \in V$ always a path from u to v or a path from v to u exists. Design an efficient algorithm which decides if a given graph G is half connected. Prove the correctness of your algorithm and analyze its time complexity.