Problem Set 3

Please hand in your solutions for this problem set via email (roesner@cs.uni-bonn.de) or personally at Room 2.060 until *Tuesday, 30th of October.*

Problem 1
We want to look at a new version of the hierarchical-kcenter algorithm. Instead of Gonzalez' algorithm we want to start with any α-approximation for the incremental clustering problem. Let $x_1, \ldots, x_{|P|}$ be the incremental solution, then we define $R_i = \max_{p \in P \setminus \{x_1, \ldots, x_{i-1}\}} d(p, \{x_1, \ldots, x_{i-1}\})$ (the radius of the incremental clustering at step $i - 1$) for $2 \leq i \leq |P|$. Afterward we use the hierarchical-kcenter algorithm as described in the lecture.

- Show that the new algorithm still yields an hierarchical clustering.

- What is, depending on α, the approximation factor of this new algorithm?

Problem 2
Recall the proof of the hierarchical-kcenter algorithm. The proof showed that the solution for any k has a radius which is at most 4 times the radius of the Gonzalez' solution.

- Create an instance for which the hierarchical-kcenter algorithm finds a solution whose radius is almost 4 times the radius of the Gonzalez' solution.

With „almost 4 times the radius of the Gonzalez' solution" we mean that we are looking for a series of instances that contains, for every $\varepsilon > 0$, an instance whose radius is at least $4 - \varepsilon$ times the radius of the Gonzalez' solution.

Problem 3
Recall the k-center-via-thresholding algorithm by Hochbaum and Shmoys.

- What is the runtime of this algorithm if it is implemented as stated in the script?

- Show how the algorithm can be adjusted to run in time $O(n^2 \log(n))$ and explain why it still works.

Problem 4

- Adjust the algorithm by Hochbaum and Shmoys to the k-supplier problem and show that it has an approximation factor of 3.