

## Advanced Algorithms

### WS 2017/18 Homework 2

23.10.2017

#### Exercise 1:

A vertex cover  $X$  of a graph  $G = (V, E)$  contains for every edge  $e \in E$  at least one of its end nodes. A vertex cover  $X \subseteq V$  is *minimum* if  $|X| \leq |X'|$  for all vertex covers  $X' \subseteq V$ . Show that in bipartite graphs the size of a maximum matching is equal to the size of a minimum vertex cover.

#### Exercise 2:

- Work out the algorithm for the computation of a maximum weighted matching in bipartite graphs in detail.
- Show that the algorithm MAXWEIMATCHING can be implemented such that its run time is  $O(n^3)$  where  $n = |A \cup B|$ .

#### Exercise 3:

Construct a weighted undirected graph with the property that a matching of maximum weight is not a matching of maximum size.

#### Exercise 4:

Let  $M_1$  and  $M_2$  be two distinct matchings of a graph  $G$  such that  $|M_1| = r$ ,  $|M_2| = s$  and  $r \leq s$ . Show that there exist two distinct matchings  $M'_1$  and  $M'_2$  such that

$$\left\lfloor \frac{s+r}{2} \right\rfloor \leq |M'_1| \leq |M'_2| \leq \left\lceil \frac{s+r}{2} \right\rceil.$$