

## Algorithmic Game Theory

Summer Term 2026

Tutorial Session - Week 5

### Exercise 1:

Consider the following single-item auction with  $n \geq 2$  bidders. The bidders simultaneously submit their bids  $b_i \geq 0$ . However, the item will always be allocated to the bidder with index 1 and the mechanism will make him/her pay the bid of the bidder with index 2.

- (a) Show that the described mechanism is truthful.
- (b) We call a mechanism *individually rational* if for all bidders  $i \in \mathcal{N}$  bidding truthfully against an arbitrary bid profile of the other players never leads to a negative utility: If  $v_i(x) \geq 0$  for all allocations  $x \in X$ , then  $u_i((v_i, b_{-i}), v_i) \geq 0$ .  
Show that the given mechanism is not individually rational.