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Due date: June 04, 2024 at 10:00 p.m.

Algorithmic Game Theory

Summer Term 2024

Exercise Set 4

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* ***task*** *which you would like to present and in* ***which of the tutorials*** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be alloecated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1: (4+4 Points)

Consider the following regret-minimization-algorithm.

Greedy

- Set $p_1^1 = 1$ and $p_j^1 = 0$ for all $j \neq 1$.
- In each round $t = 1, \ldots, T$:
  Let $L_{min}^t = \min_{i \in N} L_i^t$ for $L_i^t = \sum_{t' \leq t} \ell_i^{(t')}$ and $S^t = \{i \in N \mid L_i^t = L_{min}^t\}$.
  Set $p_i^{t+1} = 1$ for $i = \min S^t$ and $p_j^{t+1} = 0$ otherwise.

You can assume that $\ell_i^{(t)} \in \{0, 1\}$ for all i and t.

(a) Show that the costs of Greedy are at most $N \cdot L_{min}^T + (N - 1)$.

(b) State a scheme for an example such that the stated upper bound of (a) is tight for an infinite number of values T.

Exercise 2: (6 Points)

Consider a (λ, μ)-smooth game with N players and let $s^{(1)}, \ldots, s^{(T)}$ be a sequence of states such that the external regret of every player is at most $R^{(T)}$. Moreover, let s^* denote a state that minimizes the social cost. We want to upper bound the average social cost of the sequence of states. To this end, show the following bound

$$\frac{1}{T} \sum_{t=1}^{T} SC(s^{(t)}) \leq \frac{N \cdot R^{(T)}}{(1 - \mu)T} + \frac{\lambda}{1 - \mu} SC(s^*) \,.$$

Exercise 3: (2 Points)

A *fair cost-sharing game* is a congestion game such that for all resources $r \in \mathcal{R}$ the delay function can be modeled as $d_r(x) = c_r / x$ for a constant c_r.

Show that fair cost sharing games with n players are $(n, 0)$-smooth.