

## Algorithmic Game Theory

Summer Term 2024

Tutorial Session - Week 9

*You are supposed to work on these tasks in class together with your fellow students.  
Please find groups of 2 or 3 students!*

### Exercise 1:

Consider the following instance of the house-allocation problem. There are six agents  $a, \dots, f$  and their preferences are given by:

$$\begin{array}{ll} a : f > d > e > c > a > b, & b : a > c > e > f > b > d, \\ c : e > f > a > c > b > d, & d : f > a > b > c > d > e, \\ e : d > e > c > b > f > a, & f : e > a > b > c > f > d. \end{array}$$

Find a stable allocation  $\pi$  using the Top Trading Cycle Algorithm.

### Exercise 2:

In the Top Trading Cycle Algorithm initially player  $i$  owns house  $i$ . Consider a group  $S \subseteq \{1, \dots, n\}$  of players trying to cheat in the following way: The agents in  $S$  permute their houses before entering the mechanism. However, they reveal their preferences over houses truthfully.

Is there an instance where it is possible for a group  $S$  to improve at least one player in  $S$  by cheating in the above way while no other player of  $S$  gets worse? Prove your answer.