Thomas Kesselheim   June 26, 2023
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Algorithmic Game Theory

Summer Term 2023

Exercise Set 11

If you want to hand in your solutions for this problem set, please send them via email to anna.heuser@uni-bonn.de *by Tuesday evening – make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

If you would like to present one of the solutions in class, please also send an email to anna.heuser@uni-bonn.de *containing the* **task** *which you would like to present and in* **which of the tutorials** *you would like to do so. Deadline for the email is Tuesday, 10:00 pm. Please note that the tasks will be allocated via a first-come-first-served procedure, so sending this email earlier than Tuesday evening is highly recommended.*

Exercise 1:   (3 Points)
Consider the problem of Pairwise Kidney Exchange by Matching from Lecture 22. We want to show that the given matching algorithm is not DSIC if the order in which the algorithm processes the agents depends on their reports. For this purpose, consider a modified algorithm that processes agents in ascending order of node degree (tie-breaking in favor of the agent with the smallest index) and verify that there exists an instance such that there exists an agent who can do better by misreporting.

Exercise 2:   (5 Points)
Consider a set of n teams, each with 10 players, where each team owner has a ranking of all $10n$ players. Define a notion of *stable allocation* in this setting (as in Definitions 22.1 and 22.2) and show how to adapt the top trading cycle algorithm to find a stable allocation. We assume that players' preferences play no role.

Exercise 3:   (4 Points)
We call an allocation π *weakly stable* if there exists no set of agents who can obtain better houses than they are assigned in π by reallocating among themselves the houses allocated to them in π. Show that weak stability follows from stability as defined in Section 1 of Lecture 22.

Note: The converse does not hold. For example, if there are two agents who both prefer the same house, the only stable allocation is to give that house to its owner, but the alternative is also weakly stable.