

## Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 12

*As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by heart.*

### **Exercise 1:**

Show that there is not necessarily a solution to the stable roommates problem: In this problem, there is a set of  $2n$  people, each with a total preference order over all the remaining people. A matching of the people (each matched pair will become roommates) is stable if there is no pair of people that are not matched that prefer to be roommates with each other over their assigned roommate in the matching.

### **Exercise 2:**

The cake cutting protocol *cut and choose* for two agents can also be considered as a direct mechanism: Both agents simultaneously report a valuation density function  $b_i: [0, 1] \rightarrow \mathbb{R}_{\geq 0}$  for  $i = 1, 2$ . Afterwards, the mechanism cuts the point  $t$  such that  $\int_0^t b_1(x)dx = \int_t^1 b_1(x)dx = \frac{1}{2}$ . Then it allocates that piece of  $[0, t]$  and  $[t, 1]$  to agent 2 that maximizes her declared value leaving the other piece for agent 1.

Prove that the given mechanism is not DSIC.