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Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 10

As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by hard.

Exercise 1:
Consider the following single-item auctions with two bidders whose valuations are drawn independently from a uniform distribution on the interval $[0, 1]$. Do not make use of the results of Lecture 20 in order to solve subtasks (a) and (b):

(a) Show that the expected revenue of a second-price auction is $\frac{1}{3}$.

(b) Now, define a second-price auction with *reserve price* p. Let v_1 and v_2 be the valuations of the bidders. The allocation and payment rule will be determined according to the following cases:

1. $\min\{v_1, v_2\} \geq p$: Like in the second price auction.
2. $\max\{v_1, v_2\} < p$: Nobody gets the item and no payments.
3. $v_1 \geq p > v_2$: Bidder 1 gets the item and has to pay p.
4. $v_2 \geq p > v_1$: Analogous to 3.

Show that using a reserve price of $\frac{1}{2}$ the second-price auction generates an expected revenue of $\frac{5}{12}$.

Additional Task: Can you get the same results by the use of virtual values? Calculate the expected revenue of (a) and (b) using the results of Lecture 20.