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Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 9

As last week, please find yourself in groups of up to three students. Start with a quick intro-duction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by hard.
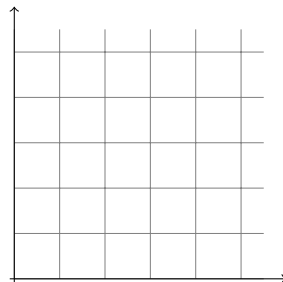
Exercise 1:
Consider three unit-demand buyers and two items a, b with

$$v_{1,a} = 5, v_{1,b} = 3, v_{2,a} = 3, v_{2,b} = 4, v_{3,a} = 2, v_{3,b} = 2 \ .$$

(a) Determine the Walrasian price vector which is determined by the VCG mechanism.

(b) Now find *all* Walrasian price vectors q. Draw these vectors in a coordinate system with axes q_a and q_b.
   As a remark, note that the prices defined by the VCG mechanism are the smallest Walrasian prices: We can use the prices from (a) as a lower bound for all Walrasian prices.

Exercise 2:
Have a look at the single-minded combinatorial auction with three bidders (red, blue, green) and items a, b, c which is depicted below. State all values of $x \in \mathbb{R}_{\geq 0}$ such that there exists a Walrasian equilibrium and prove your claim.