

## Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 8

*As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by heart.*

### Exercise 1:

Consider the following single-item auction: Each bidder reports a bid  $b_i \geq 0$ . The bidder with the highest bid wins the item and pays *half* his bid.

- (a) Show that if we only consider two bidders and valuations are drawn uniformly from  $[0, 1]$ , then truthful bidding is a Bayes-Nash equilibrium.
- (b) Show that this mechanism is not dominant-strategy incentive compatible.
- (c) Show that this mechanism is  $(\frac{1}{2}, 1)$ -smooth.

### Exercise 2:

Recall the auction of  $k$  identical items from the previous exercise sets. Bidder  $i$  has value  $v_i$  if he/she gets at least one of the items, 0 otherwise. We define a mechanism as follows: the bidders who reported the  $k$  highest bids win an item. Each of them has to pay their respective bids. Show that if losers (i.e. bidders who do not get any item) do not pay anything, this mechanism is  $(\frac{1}{2}, 1)$ -smooth.