

## Algorithmic Game Theory

Summer Term 2023

Tutorial Session - Week 6

As last week, please find yourself in groups of up to three students. Start with a quick introduction. Afterwards, you are supposed to discuss the exercises on this sheet and in addition talk about definitions, proof ideas and techniques used in the lecture. Also, feel free to open the lecture notes and have a look if you don't remember a certain definition or theorem by heart.

### Exercise 1:

Consider the following single-item auction with  $n \geq 2$  bidders. The bidders simultaneously submit their bids  $b_i \geq 0$ . However, the item will always be allocated to the bidder with index 1 and the mechanism will make him/her pay the bid of the bidder with index 2.

- (a) Show that the described mechanism is truthful.
- (b) We call a mechanism *individually rational* if for all bidders  $i \in \mathcal{N}$  bidding truthfully against an arbitrary bid profile of the other players never leads to a negative utility: If  $v_i(x) \geq 0$  for all allocations  $x \in X$ , then  $u_i((v_i, b_{-i}), v_i) \geq 0$ .

Show that the given mechanism is not individually rational.

### Exercise 2:

Analogous to the auctions that we defined in the lecture, we will consider the following *Third-Price Auction*. Just like in the first- and second-price auctions, bidders simultaneously submit their bids  $b_i \geq 0$  and the winner will be determined as the bidder with the highest bid. Finally, the mechanism will make him/her pay the third highest bid. Prove that the described mechanism is not truthful.