Thomas Kesselheim
Alexander Braun

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Algorithms and Uncertainty

Summer Term 2021

Tutorial Session - Live Tasks 10

Exercise 1:
For the normed vector space $(\mathbb{R}^d, \|.\|)$, the unit ball with respect to $\|.\|$ is defined as the set $\{x \in \mathbb{R}^d : \|x\| \leq 1\}$.

(a) Show that the unit ball with respect to the 1-norm is convex.

(b) Show that the unit ball with respect to the 2-norm is convex.

(c) Show that the unit ball with respect to the ∞-norm is convex.

(d) Does the same result hold for an arbitrary p-norm with $p > 1$? What about $p < 1$?

Exercise 2:
Consider the following problem motivated by web search: Suppose there are T users that all search for the same keyword. There are k different results that they might be interested in. Whenever a user arrives, we display these k results in an order that we choose. Afterwards we get to know which of the k results the user was interested in and incur a cost of j if this was the j^{th} result in our order.
Model this problem as an online convex optimization problem so that Follow the (Regularized) Leader can be applied.