Thomas Kesselheim
Alexander Braun

July 1, 2021

Algorithms and Uncertainty

Summer Term 2021

Tutorial Session - Live Tasks 9

Exercise 1:
Show that every no-regret algorithm in the experts setting has to be randomized. Consider the case $n = 2$ and for every deterministic algorithm construct a sequence such that $L_{\text{Alg}}^{(T)} = T$ and $\min_i L_i^{(T)} \leq \frac{T}{2}$.

Exercise 2:
Let each $l_i^{(t)} \in \{0, 1\}$. We consider the following Greedy algorithm. In each step t, the algorithm selects I_t which satisfies $I_t = \arg\min_{i \in [n]} L_i^{(t-1)}$, i.e. the expert with the best cumulative cost so far (ties are broken adversarially).

(a) Show that $L_{\text{Alg}}^{(T)} \leq n \cdot \min_i L_i^{(T)} + (n - 1)$

(b) Is the result of (a) surprising? Argue by the use of an appropriate lower bound.