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Algorithms and Uncertainty

Summer Term 2021

Tutorial Session - Live Tasks 8

Exercise 1:
Recall the regret definition from the lecture: $\text{Regret}^{(T)} = L_{\text{Alg}}^{(T)} - \min_i \sum_{t=1}^{T} \ell_i^{(t)}$.
We want to understand the order of minimum and sum in the second term. Therefore, work on the following tasks.

(a) Use Yao's principle to show that for every (randomized) algorithm there is a sequence $\ell^{(1)}, \dots, \ell^{(T)}$ such that $L_{\text{Alg}}^{(T)} \geq \left(1 - \frac{1}{n}\right) T$ but $\sum_{t=1}^{T} \min_i \ell_i^{(t)} = 0$.

(b) Discuss the importance of the order of sum and minimum in the regret definition using your results from (a).

Exercise 2:
Consider the modified update rule for Multiplicative Weights that sets $w_i^{(t+1)} = w_i^{(t)} \cdot (1 - \ell_i^{(t)} \eta)$. Show that Theorem 19.3 still holds.