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Due: July 21, 2021 at noon

Algorithms and Uncertainty

Summer Term 2021

Exercise Set 11

Exercise 1: (3 Points)
Prove Observation 24.4: If R is σ-strongly convex and $f_1, f_2, \dots$ are convex then $R + \sum_t f_t$ is σ-strongly convex.

Exercise 2: (4 Points)
We consider Online Linear Regression as introduced in the lecture. Recall that

$$f_t(w_1, w_2) = \left(w_1 x^{(t)} + w_2 - y^{(t)}\right)^2 \quad .$$

Derive a regret bound for Follow-the-Regularized-Leader with Euclidean regularization under the assumption that $|x^{(t)}|, |y^{(t)}| \leq 1$ for all t and $S = \{\mathbf{w} \in \mathbb{R}^2 \mid \|\mathbf{w}\|_2 \leq r\}$.

Exercise 3: (4 Points)
Derive a regret bound for Follow-the-Regularized-Leader if the Lipschitz constant depends on the time step, that is,

$$f_t(\mathbf{u}) - f_t(\mathbf{v}) \leq L_t \|\mathbf{u} - \mathbf{v}\| \quad \text{for all } \mathbf{u}, \mathbf{v} \in S \ .$$