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Algorithms and Uncertainty

Summer Term 2021

Exercise Set 2

If you want to hand in your solutions for this problem set, please send them via email to alexander.braun@uni-bonn.de - *make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible.*

Exercise 1: (4 Points)
Given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the *frequency* of the set system. Now, we generalize the algorithm from Tutorial 2, Exercise 2 to the weighted version. Let $g_e = \max\{0, 1 - \sum_{S:e \in S} x_S\}$ and let S_e be the cheapest set covering e. For each S that covers e, increase x_S by $\frac{c_{S_e}}{c_S} g_e$ and set $y_e = c_{S_e} g_e$. Show that this algorithm is f-competitive by using Lemma 3.7.

Exercise 2: (5 Points)
Again, given an instance of Set Cover, let $f = \max_{e \in U} |\{S \in \mathcal{S} \mid e \in S\}|$ denote the frequency of the set system.
Use our results from the fourth lecture to devise an online algorithm that is $O(\log f)$-competitive for fractional set cover and prove this. You may assume that f is known beforehand.

Hint: One bound in the analysis from the lecture can be improved for $f < n$. Use it to adapt the algorithm.