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Due: April 21, 2021 at noon

Algorithms and Uncertainty

Summer Term 2021

Exercise Set 1

If you want to hand in your solutions for this problem set, please send them via email to alexander.braun@uni-bonn.de - *make sure to send a pdf-file which contains your name and your email address. Of course, submitting solutions in groups is also possible. We will discuss this sheet in the tutorials on April 22.*

Exercise 1: (1+3 Points)

Consider the following algorithm for the ski rental problem: Buy a pair of ski on your first skiing day for a price of B.

(a) Give a sequence σ of skiing/non-skiing days such that $c(\mathrm{ALG}(\sigma)) = c(\mathrm{OPT}(\sigma))$.

(b) Is there an $\alpha > 0$ such that this algorithm is strictly α-competitive? If yes, give a proof, otherwise a counterexample.

Exercise 2: (3 Points)

The Online Bipartite Vertex Cover problem is defined as follows: We are given a bipartite graph with vertices $V = L \cup R$. The nodes in L are offline which means they are present initially. Nodes in R are online and revealed one at a time together with its incident edges. Each vertex $v \in V$ has a cost c_v. We need to maintain a feasible vertex cover in every step with the goal of minimizing the overall incurred costs.

Consider the ski rental problem in a simplified version, i.e. we assume that every day is a skiing day but we do not know the number of days in advance. Show in a constructive way that the ski rental problem is a special case of the Online Bipartite Vertex Cover problem.

Exercise 3: (4 Points)

We want to show that the assumptions (completeness and triangular inequality) in the lecture concerning the online Steiner tree problem are indeed without loss of generality. Therefore, consider an α-competitive online algorithm for the online Steiner tree problem on complete graphs satisfying the triangular inequality (a.k.a. metric Steiner tree problem). Show how to convert this algorithm into an α-competitive one for the general online Steiner tree problem.