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Due date: In the lecture on 29th June 2019, 12:15

Algorithmic Game Theory and the Internet

Summer Term 2019

Exercise Set 8

Exercise 1: (4 Points)

Recall the *Greedy-by-Value* and *Greedy-by-Sqrt-Value-Density* algorithms for single-minded CAs of lecture 12. Let us analyse another greedy algorithm that looks as follows.

Greedy-by-Value-Density

- Re-order the bids such that $\frac{b_1^*}{|S_1^*|} \geq \frac{b_2^*}{|S_2^*|} \geq \cdots \geq \frac{b_n^*}{|S_n^*|}$.
- Initialize the set of winning bidders to $W = \emptyset$.
- For $i = 1$ to n do: If $S_i^* \cap \bigcup_{j \in W} S_j^* = \emptyset$, then $W = W \cup \{i\}$.

Let $d = \max_{i \in \mathcal{N}} |S_i^*|$. Show that the given algorithm yields a d-approximation.

Exercise 2: (3 Points)

Recall the auction of k identical items from Exercise Set 6. Each bidder can acquire at most one of the items. If bidder i gets one of the items, she has a value of v_i. Otherwise, that is, if she does not get an item, she has a value of 0. Make use of the VCG-results from the lecture in order to design a truthful mechanism for this auction. For this purpose, explicitly state the function f and calculate the payment rule p.

Exercise 3: (3 Points)

Consider a single-parameter problem and let f be the function that maximizes $\sum_i b_i x_i$ among all $x \in X$ (declared welfare). Show that f is monotone.

Exercise 4: (4 Points)

As seen in the lecture, let $f \colon V \to X$ be a function that maximizes declared welfare, i.e., $f(b) \in \arg\max_{x \in X} \sum_i b_i(x)$ for all $b \in V$. For each i, let h_i be an arbitrary function $b_{-i} \mapsto h_i(b_{-i})$ which does not depend on b_i. We define a mechanism $M = (f, p)$ by setting

$$p_i(b) = h_i(b_{-i}) - \sum_{j \neq i} b_j(f(b)) \ .$$

Prove that M is a truthful mechanism.

Exercise 5 on the next page.

Exercise 5: (4+2 Points)

Consider the following *Procurement Auction*. It's being attempted to buy a certain item. There are n vendors who are able to manufacture the wanted item. Vendor i incurs a cost of c_i for crafting the item. Now, the vendors are asked to state their costs for crafting the item and a vendor with lowest cost shall be chosen. The latter potentially gets a payment for it. The stated problem can be formalized by the general model of the lecture: Each vendor i is interpreted as a bidder who has negative valuation v_i, if he/she is chosen to craft the item, that is, $v_i(x) = -c_i$, if i is chosen in x.

(a) The results of the lecture concerning VCG are applicable in this situation. Make use of them in order to state a truthful mechanism. Note that this mechanism won't be *individually rational*.

(b) Make use of the results from Exercise 4 in order to modify the mechanism to be individually rational.