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Due date: In the lecture on 24th April 2019, 10:15

Algorithmic Game Theory and the Internet

Summer Term 2019

Exercise Set 3

Exercise 1: (3+3+4 Points)

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) which is defined the following way:

Instances: Propositional logic formula with n binary variables $x_1, \ldots, x_n$ that is described by m clauses $c_1, \ldots, c_m$. Each clause c_i has a weight w_i and consists of exactly k literals, which are all positive (i.e., the formula does not contain any negated variable $\overline{x}_i$).

Feasible solutions: Any variable assignment $s \in \{0, 1\}^n$

Objective function: Sum of weights of clauses c_i in which not all literals are mapped to the same value.

Neighbourhood: Assignments s and s' are *neighbouring* if they differ in the assignment of a single variable.

(a) Show: Pos-NAE-kSAT is in PLS.

(b) Show: Pos-NAE-2SAT $\leq_{PLS}$ MaxCut

(b) Show: Pos-NAE-3SAT $\leq_{PLS}$ Pos-NAE-2SAT

Exercise 2: (4 Points)

We define a Congestion Game to be *symmetric*, if $\Sigma_1 = \ldots = \Sigma_n$. Let $PNE_{\text{Cong. Game}}$ and $PNE_{\text{Sym. Cong. Game}}$ be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show: $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$.

Hint: Add an auxiliary resource for each player with a suitable delay function.

Exercises 3 and 4 on the next page.

Exercise 3: (2+2 Points)

Consider the following cost-minimization game. Two car drivers approach a junction. Both drivers can either stop at (S) or cross (C) the junction. If a driver decides to stop, small costs emerge to her because of the waiting time. If both drivers decide to cross the junction, they will crash – resulting in high costs for both drivers.

	C(ross)		S(top)	
C(ross)		100		1
	100		0	
S(top)		0		1
	1		1	

(a) List all pure and mixed Nash equilibria.

(b) State a *coarse-correlated equilibrium* that is not a pure or mixed Nash equilibrium.

Hint: Think of a probability distribution p "implementing" traffic lights.

Exercise 4: (2 Points)

Let p, p' be coarse correlated equilibria of a cost-minimization game Γ. Prove that any convex combination of the distributions p and p' yields also coarse correlated equilibrium of Γ (i.e., any distribution $q := \lambda p + (1 - \lambda)p'$ for a $\lambda \in [0, 1]$).