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Complexity of Boolean functions
SS 2019
Homework 7

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Exercise 1:

Develop an algorithm which computes for each node g of a given monotone network β the formulas $\mathrm{DNF}_\beta(g)$ and $\mathrm{CNF}_\beta(g)$.

Exercise 2:

a) Describe a CNF/DNF-switch.

b) Let α be a DNF-formula (CNF-formula). Prove that the formula γ obtained by a DNF/CNF-switch (CNF/DNF-switch) computes the same function as α.

Exercise 3:

Consider the lower bound proof for the clique function of the lecture.

a) Show that the number of inputs in T_1 for which $\mathrm{DNF'}_\beta(g_i)$ could introduce an error is bounded by $\binom{m-r}{s-r}\left(\frac{m}{4s}\right)^r$.

b) Show that the number of inputs in T_0 for which $\mathrm{CNF'}_\beta(g_i)$ could introduce an error is bounded by $\left(\frac{s}{2}\right)^k (s-1)^{m-k}$.

c) Show that either $\mathrm{CNF'}_\beta(g_t)$ computes the constant function one or $\mathrm{CNF'}_\beta(g_t)$ computes the value of at least half of the inputs in T_1 incorrectly.

Exercise 4:

Prove Theorem 5.1 of the lecture.