

## Algorithmic Game Theory and the Internet

### Summer Term 2018

#### Exercise Set 3

##### **Exercise 1:** (4+4+4 Points)

Consider the local search problem *Positive Not-All-Equal kSat* (Pos-NAE-kSAT) which is defined the following way:

**Instances:** Propositional logic formula with  $n$  binary variables  $x_1, \dots, x_n$  that is described by  $m$  clauses  $c_1, \dots, c_m$ . Each clause  $c_i$  has a weight  $w_i$  and consists of exactly  $k$  literals, which are all positive (i.e., the formula does not contain any negated variable  $\bar{x}_i$ ).

**Feasible solutions:** Any variable assignment  $s \in \{0, 1\}^n$

**Objective function:** Sum of weights of clauses  $c_i$  in which not all literals are mapped to the same value.

**Neighbourhood:** Assignments  $s$  and  $s'$  are *neighbouring* if they differ in the assignment of a single variable.

- (a) Show: Pos-NAE-kSAT is in PLS.
- (b) Show: Pos-NAE-2SAT  $\leq_{PLS}$  MaxCut
- (b) Show: Pos-NAE-3SAT  $\leq_{PLS}$  Pos-NAE-2SAT

##### **Exercise 2:** (4 Points)

We define a Congestion Game to be *symmetric*, if  $\Sigma_1 = \dots = \Sigma_n$ . Let  $PNE_{\text{Cong. Game}}$  and  $PNE_{\text{Sym. Cong. Game}}$  be the local search problems in PLS of finding a pure Nash equilibrium of a general or symmetric Congestion Games, respectively.

Show:  $PNE_{\text{Cong. Game}} \leq_{PLS} PNE_{\text{Sym. Cong. Game}}$ .

**Hint:** Add an auxiliary resource for each player with a suitable delay function.

**Exercise 3:**

(2+2 Points)

Consider the following cost-minimization game. Two car drivers approach a junction. Both drivers can either stop at (S) or cross (C) the junction. If a driver decides to stop, small costs emerge to her because of the waiting time. If both drivers decide to cross the junction, they will crash – resulting in high costs for both drivers.

		C(ross)	S(top)
		100	1
C(ross)	100	0	
	S(top)	1	1

- (a) List all pure and mixed Nash equilibria.
- (b) State a *coarse-correlated equilibrium* that is not a pure or mixed Nash equilibrium.

**Hint:** Think of a probability distribution  $p$  “implementing” traffic lights.