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Complexity of Boolean functions
SS 2018
Homework 9

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Exercise 1:

a) Show that Claim 1 and Claim 2 at page 161 of the lecture notes imply the lower bound $\frac{1}{6}(\frac{q}{r-1})^{s/2}$.

b) Prove Corollary 4.2 of the lecture.

Exercise 2:

Develop an algorithm which computes for each node g of a given monotone network β $\mathrm{DNF}_\beta(g)$ and $\mathrm{CNF}_\beta(g)$.

Exercise 3:

a) Describe a CNF/DNF-switch.

b) Let α be a DNF-formula (CNF-formula). Prove that the formula γ obtained by a DNF/CNF-switch (CNF/DNF-switch) computes the same function as α.

Exercise 4:

Consider the lower bound proof for the clique function which uses DNF/CNF-approximators.

a) Show that the number of inputs in T_1 for which the approximator D_g^r could introduce an error is bounded by $\binom{m-r}{s-r}(\frac{m}{4s})^r$.

b) Show that the number of inputs in T_0 for which the approximator C_g^k could introduce an error is bounded by $(\frac{s}{2})^k(s-1)^{m-k}$.

c) Show that either $C_{g_0}^k$ computes the constant function one or $C_{g_0}^k$ computes the value of at least half of the inputs in T_1 incorrectly.

d) Prove Theorem 5.1 of the lecture.