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Randomized Algorithms and
Probabilistic Analysis
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Problem Set 7

Problem 1

Assume that we throw $m = \lfloor 5n \log_2 n \rfloor$ balls into n baskets. For each ball, the basket is chosen uniformly at random. Let X_i be the number of balls in basket i for $i \in \{1, \ldots, n\}$, and set $X = \max_{i=1,\ldots,n} X_i$. Use Thm 3.9 to show that

$$\mathbf{Pr}(X \geq 30 \log_2 n) \leq \frac{1}{n^c}$$

holds for a suitable constant $c > 0$.

Problem 2

Show that $\binom{n}{k} \in \Theta(n^k)$ if $k \in \mathbb{N}$ is a constant. Is this still true if $k = n/2$ (and thus not a constant?).

Problem 3

Let $a \in \{0,1\}^n$ be uniformly chosen from all possible assignments $\{0,1\}^n$, and let a^* be an unknown but fixed assignment. Let $r := |\{i \in \{1, \ldots, n \mid a_i = a_i^*\}|$ be the number of positions where a and a^* agree. Show that

$$\Pr(r \geq n/2) \geq 1/2.$$

Problem 4

For the following discrete random variables decide which of Markov's inequality, Chebyshev's inequality and Chernov/Rubin bounds you can apply and use the applicable ones to bound the probability $Pr(X \geq a)$.

- Let $\Omega = \mathbb{N}$ and $k = \sum_{i=1}^{\infty} \frac{1}{i^3}$. Let $Pr(X = i) = \frac{1}{ki^3}$ and let $a = 2$.
- You play a game where you need to guess a date and time (hours and minutes). For simplicity we assume that the calender in the game has exactly 30 days in each month. Independent of each other you get a point for each of the following you have guessed correctly: day, month, day of the week, hour, minute, am/pm. Let X denote the number of points obtained in the game and let $a = 1.5$.