- *You can hand in your written solutions until Tuesday, 03.12., 14:15, in room E.06.*

Exercise 16:   **Lower envelopes**   (4 points)

Let L be a set of n non-vertical lines (given by equations $y = a_i x + b_i$) and P be a set of n parabolas (given by equations $y = c_i(x - d_i)^2 + e_i$), where $a_i, b_i, c_i > 0, d_i$ and e_i are constants.

Prove that the lower envelope of L consists of at most n line segments and that the lower envelope of P consists of at most $2n - 1$ arcs.

Exercise 17:   **Two-Ears Theorem**   (4 points)

Given a triangulation T of a simple polygon P, an *ear* of P is a triangle of T of which at least two edges are also boundary edges of P. Show that if P has at least four vertices, then given any triangulation T of P, it holds that P has at least 2 ears.

Exercise 18:   **Colouring triangulations**   (4 points)

1. Prove that if P is the vertex set of a simple polygon P', and T is a triangulation of P', then T is 3-colourable.

2. Prove or disprove that if P is an arbitrary (finite) point set in the plane, and T is a triangulation of P, then T is 3-colourable.